

Sampling Signals

Bustec Production Ltd.

Contents

1	Sampling Signals (Analog and Digital)	1
1.1	Sampling Theorem	1
1.2	Signal Aliasing	2
1.3	Signal Aliasing and Wide-band Noise	3
1.4	Filtering Aliased Signals	4
1.5	Sample Time Jitter	4

1. Sampling Signals (Analog and Digital)

By sampling a signal we are attempting to represent the time dependence of the signal by a discrete set of samples. As a practical convenience we typically depend on a fixed sample interval. The mathematical basis for most data analysis critically depends on an underlying fixed sample interval. While our primary focus will be the sampling and digitizing of continuous analog signals, most of the principles discussed apply equally to sampling of digital signal streams.

It is important to realize that in all cases we are attempting to represent the information in a time varying signal by a set of discrete samples. In the process of sampling we are losing information. Obviously we lose information regarding signal variations shorter than the sampling interval.

1.1. Sampling Theorem

The sampling theorem simply stated says that one must sample a signal at a *minimum* of twice the highest frequency of interest. Thus to represent a signal with frequency components between zero and 1 KHz one must sample the signal at 2 KHz or above.

As a practical matter where one is observing the time dependence of a signal directly, one may choose to sample at a rate of four times the highest frequency or more so as to have multiple samples over the period of the highest frequency.

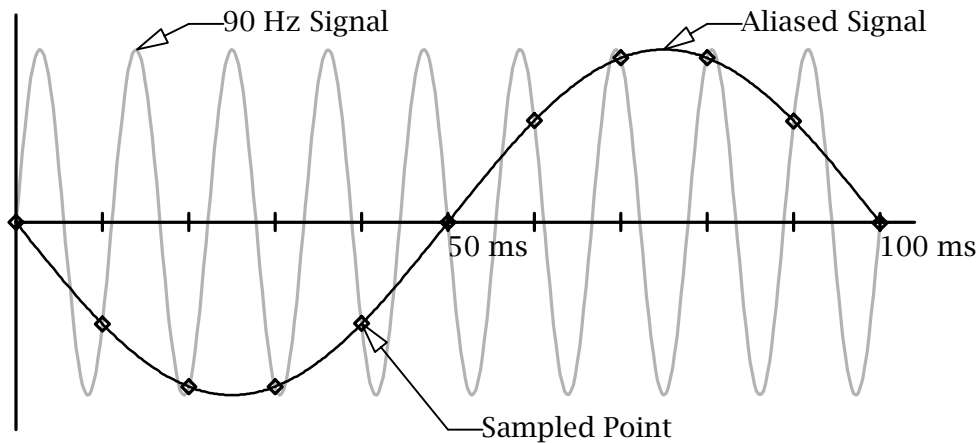


Figure 1: Signal alias example of 90 Hz sine wave sampled at 100 Hz

1.2. Signal Aliasing

Signal aliasing is a phenomena that occurs when one samples a signal that contains frequency components above the Nyquist frequency as given by the sampling theorem i.e. above half the sampling rate. The effect can be easily understood from Figure 1. In this example a 90 Hz sine wave signal is sampled at 100 Hz. The *aliased* signal inferred from the samples is a 10 Hz sine wave (100 ms period). *In a real data system it is the samples that one observes.*

It is worth noting here that had the original 90 Hz signal been a discrete set of 100 samples acquired at a sample rate of 1KHz and that only every 10th sample was kept, the resulting signal aliasing would have been the same as the analog case above. It doesn't matter whether the *sample* is an analog value or a digitized value. When down-sampling (decimating) digital data it is important to first limit the digital signal bandwidth *prior* to down-sampling to a value consistent with the Nyquist frequency of the new (down-sampled) sampling rate.

In a real data system the effects of signal aliasing can be quite important. If there are frequency components, including noise, present in the original signal with frequencies above the Nyquist frequency f_n (one half the sampling rate) then these frequencies will alias into the observed frequency band from 0 to f_n . The net effect is to distort the inferred time dependence of the signal based on the observed data samples as well as the signal frequency components inferred from the sampled data. In the example illustrated in Figure 1 one would infer a strong 10 Hz frequency component in the sampled data when in-fact it is an aliased 90 Hz signal.

For example consider a situation where one is sampling a signal at 100 Hz which

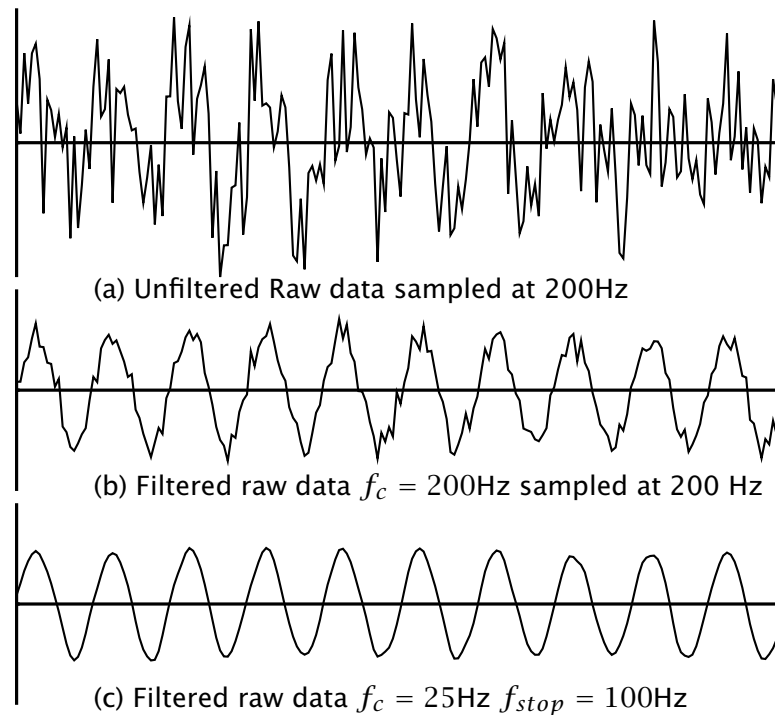


Figure 2: 10Hz Sine wave with wide-band (10KHz) noise

contains a small amount of 60 Hz power line pickup. The 60 Hz pickup would appear as 40 Hz “noise” in the sampled signal.

1.3. Signal Aliasing and Wide-band Noise

Most instrumentation today is designed to handle a wide range of sample rates and signal bandwidths. Typical instrumentation amplifiers have bandwidths extending well above 10 to 100 KHz. While signals of interest may have considerably lower bandwidths and sampling rate requirements, noise up to the full bandwidth can be present. In such cases wide-band noise can alias into the passband resulting in poor signal-to-noise in the digitized data.

Consider the situation where a low-level signal is sampled at 200 Hz in the presence of low-level wide-band white noise up to 10 KHz. The white noise between 100–200, 200–300, ..., 9900–10,000 Hz is aliased into the 0–100 Hz region significantly increasing the noise level in the observed sampled data. Figure 2 illustrates this effect. The top trace (a) is the result of sampling a channel with approximately

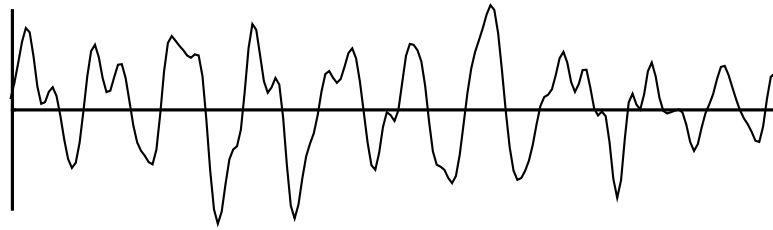


Figure 3: Filtering of signal after down-sampling

equal amounts of a 10Hz sine wave and white noise extending from 0 to 10KHz.

The middle trace in Figure 2(b) results when the raw 10KHz bandwidth signal is first passed through a lowpass filter prior to sampling. The filter has a cutoff frequency of $f_c = 200\text{Hz}$ that effectively eliminates noise with frequency components above 800Hz. Note that this does not eliminate aliasing of noise below 800Hz.

The bottom trace in Figure 2(c) results when the filter cutoff is reduced to $f_c = 25\text{Hz}$ and a stopband edge at 100Hz. This eliminates aliasing and we are left with the 10Hz sine wave and the noise below 100Hz—a huge improvement over no filtering.

1.4. Filtering Aliased Signals

It is important to note here that it is not possible to undo the effects of aliasing once a signal with frequency components above the Nyquist frequency has been sampled. Figure 3 illustrates the result of filtering after the signal is aliased. It is the result of filtering the down-sampled signal illustrated in Figure 2(a). The observed improvement is due to the partial attenuation of frequency components between the filter cutoff at 25Hz and 100Hz.

Low-pass filters to limit signal bandwidth to below the Nyquist frequency must be located prior to the point where the signal is effectively sampled. In analog-to-digital converters where multiple signals are sequentially multiplexed and digitized the signal is effectively sampled at the multiplexer. In this case antialias and noise filtering needs to occur ahead of the multiplexer to be effective.

1.5. Sample Time Jitter

Another important phenomena is the effect of jitter in the sample time interval. Almost all data analysis techniques are predicated on a series of equally spaced samples in time. One way to look at sample interval jitter δt is to compute the worst case *measurement error* δV that results from sampling a full system bandwidth sine wave $V_0 \sin(\omega t)$.

$$dV/dt = \omega V_0 \cos(\omega t)$$

$$dV/dt = \omega V_0 \dots(\text{worst case})$$

$$\delta V/V_0 = 2\pi f \delta t$$

$$\text{or } (\delta V/V_0) = 6.28(\delta t/T) \text{ Where } T \text{ is the period}$$

From this it should be clear that a jitter in the sampling interval translates directly into a measurement error that is proportional to the frequency of the signal times the amount of jitter.

An important feature of any data system that samples analog data is the requirement for a stable noise-free sample clock at the point that data is sampled.

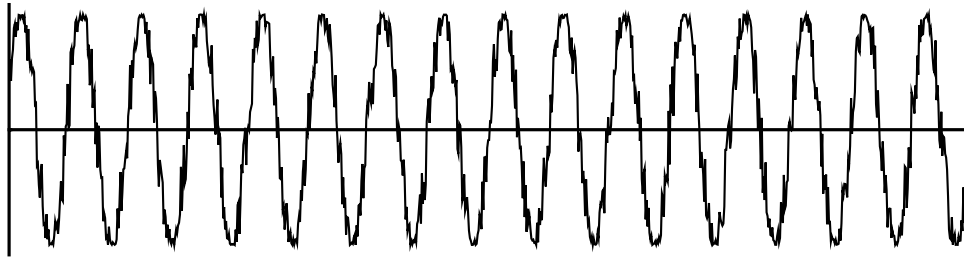


Figure 4: Time domain effects of random sample interval jitter on a sine wave signal.

The time domain effects of sample interval jitter are illustrated in Figure 4. The graph illustrates the effects of a 10% random sample interval jitter on the sampling of a pure sine wave in the time domain.

Figure 5 illustrates the effects of sample interval jitter on the noise floor in the frequency domain for both a 10% and 0.1% sample interval jitters.

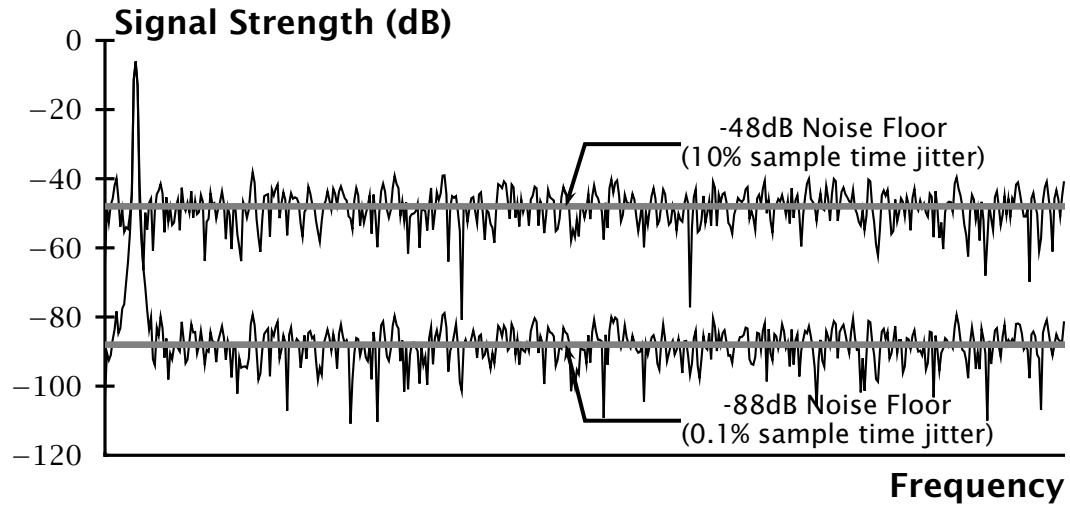


Figure 5: Frequency domain effects of random sample interval jitter on a sine wave signal.

Corporate Headquarters:

Bustec Production Ltd.
World Aviation Park
Shannon, Co. Clare
Ireland
Tel: +353 61 707 100
Fax: +353 61 707 106

Bustec, Inc.
17820 Englewood Dr. #14
Middleburg Hts. OH 44130
Tel: (440) 826 4156
Fax: (440) 826 4184
